Macroeconomic Adjustment to Monetary Union*

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Abstract

The move to monetary union in Europe led to convergence of interest rates among the participating countries. This was associated with notable cross-country differences in the behaviour of key macroeconomic aggregates. Compared to the low interest rate countries, former high interest rate countries experienced a boom in domestic demand, a deterioration of the current account and appreciation of the real exchange rate. This paper documents the key stylised facts of this experience and provides a compact two-country model, based on the Blanchard-Yaari setup, to analyze this phenomenon. This model, though simple, is able to broadly capture the main qualitative features of the adjustment. Using this model, we show that the creation of the monetary union leads to an increase in welfare for all generations in both country groups.

1 Introduction

The creation of the euro area in 1999, a key landmark in the history of European integration, is likely to have had important effects on the economies of the participating countries.[1]

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[1]Participation in the monetary union is expected to affect trade and financial integration, business cycle synchronization and the patterns of specialization. It could also affect institutions, and the structure and

In this paper, we focus on the macroeconomic implications of one crucial aspect of monetary union, namely increased financial integration. The most obvious manifestation of this integration was the convergence of short and long-term interest rates to the relatively low levels prevailing in Germany. Against this background, we address three questions. First, how did the macroeconomic adjustment to monetary union differ between former high-interest rate and low interest rate countries? Second, can we develop a compact general equilibrium model to account for these facts? Third, what are the welfare implications of the creation of the monetary union for the respective countries?

As regards the first question, building on our earlier work, (Fagan and Gaspar (2007)), we try to establish some key stylised facts regarding the macroeconomic effects of interest rate convergence. We compare the behaviour over time of a series of macroeconomic variables for two groups of countries: core countries (characterized by low interest rates before participation in the euro area) and convergence countries (characterized by high interest rates). We use two complementary methods for this purpose: a graphical comparison of cross-country averages and the difference-in-difference estimator. On the basis of this analysis, we show that there were with notable cross-country differences in the behaviour of key macroeconomic aggregates: domestic demand, current account balances and intra-euro area real exchange rates. Compared to the core countries, the converging countries experienced a boom in domestic demand, a deterioration of the current account and appreciation of the real exchange rate. An important feature, which has a major influence on our modelling approach, is that the convergence of interest rates was not associated with significant changes in output growth differentials.

Second, we develop a compact two-country dynamic general equilibrium model with a view to matching these facts[2]. We focus on an endowment economy setup with two goods (traded and non-traded). The impulse we consider involves the creation of a monetary union and resulting financial integration of two (groups of) countries which were previously characterised by different levels of domestic interest rates. In our setup, as explained below, the initial difference in interest rates will be assumed to reflect fundamental differences between the two economies, specifically differences in the rate of time preference. After

behavior of product and labor markets (see Lane (2006) and Mongelli and Vega (2006) for comprehensive surveys of the relevant issues and available evidence).

[2]In earlier papers (Fagan and Gaspar (2005) and Fagan and Gaspar (2007)), we developed models to analyse the effects of interest rate convergence from the perspective of a small open economy entering the monetary union and experiencing an exogenous permanent fall in interest rates. This approach suffers from a very important limitation: it assumes that the former high-interest rate countries can be characterised as small open economies within the euro area. This assumption is strongly counterfactual. In fact, the converging countries account for some 37% of euro area GDP. Hence, it is clear that significant feedback effects cannot be excluded *a priori*. Moreover, the adjustment in the countries that previously enjoyed low interest rates is interesting as well.

the monetary union, a single interest rate (which may vary over time) exists in the two countries. We then use our model to examine the macroeconomic affects of interest rate convergence. We will argue that such a simple framework is capable of capturing most salient, qualitative features of adjustment mentioned in the previous paragraph.

Third, using the model developed in this paper, we derive the welfare implications of the creation of the monetary union. We show that the creation of the monetary union is Pareto improving: all generations in both countries experience a gain in welfare. This reflects the fact that the monetary union represents an expansion of the opportunity sets of all agents in the model. Using a calibrated version of the model, we find that the gains in welfare are greater for later generations and for agents in the former higher interest rate countries.

The paper is organized as follows. In Section 2, we review some stylized facts in order to motivate our analysis. In Section 3, we present our model. Sections 4 and 5 present analytical results for the model in the case respectively of financial autarky and monetary union. Section 6 presents the results of numerical simulations of the model. In Section 7, we explore the welfare implications of the creation of the monetary union while Section 8 concludes.

2 Stylised facts.

The establishment of the monetary union in Europe in 1999 was preceded by a process of nominal convergence among the participating countries. In 1996, for example, nominal long term interest rates ranged from a high of 14.5 percent in Greece to 4.5 percent in Germany. Currently the spread between the highest and lowest government bond yields is of the order of 30 basis points. Similarly, the highest-lowest spread in short-term interest rates amounted to 10.5 percentage points in 1996. Currently, this spread is effectively zero, reflecting the complete integration of money markets in the euro area. Of course, a significant component of the convergence in nominal interest rates across countries reflected a convergence of expected inflation. Still, the evidence shows that real interest rates were also significantly affected (see Charts 1 to 3). Broadly speaking it may be said that monetary unification was associated with convergence of financial conditions across member states.

What were the economic effects of the decline in interest rates differentials (or of the convergence of financial conditions) on the relative behaviour of the countries concerned? Clearly there are too many factors, some country-specific, varying over time to be able to conclude much from individual country data unless a detailed modelling structure is

employed[3]. Key macroeconomic variables in each of the countries were affected by a host of specific shocks, including common shocks emanating from the global environment (e.g. changes in world demand, world interest rates and oil prices) as well as country specific shocks. A more promising approach, is to look at differentials in the behaviour of variables across countries over time. However, differentials across country pairs would still be effected by country specific shocks, so it is difficult to identify the effect of the shock in this manner. However, taking (simple) averages of the country pairs should reduce the importance of country specific factors and offers some chance of highlighting the common adjustment mechanism. Following our earlier work (Fagan and Gaspar (2007)) we start by separating the euro area countries into two groups on the basis of the relative nominal and real interest rates prevailing in the mid 1990s. The first group comprises low interest rate countries Germany, Belgium, France, the Netherlands and Austria[4]. In common with the usual parlance, we denote this group as the core countries. The second group comprises countries which, initially, had relatively high interest rates: Spain, Ireland, Italy and Portugal[5]. We call this group the converging countries. We calculate, for each economic variable, a pair of time series based on arithmetic (equally weighted) averages within the groups. Then, with a view to controlling for the possible effects of common shocks across all countries, we compare the differences across these averaged country groups. We display the results graphically.

We complement this graphical approach with a more rigorous method. Specifically, we employ a difference-in-difference estimator. This method is often used to study the impacts of one-off events, such as a medical treatments or the effects of specific policies, such as training programs (Ashenfelter (1978)) or minimum wages (Card and Krueger (1994)).

The idea is that we have data on two periods (the pre-EMU and the post-EMU periods) for two groups of countries (the converging and the core group). By comparing the change in the difference between the two groups of countries over the two periods one may derive an estimate of the differential effect of participation in the euro area on the two country groups. The null hypothesis in this approach is that in the absence of the financial integration

[3]Using a DSGE model, Langedijk and Roeger (2007) examine the experience of 6 euro area countries - Germany, Ireland, Italy, Spain, Portugal and the Netherlands - in the early years of EMU. They find that a large part of the divergence between these countries can be explained by the one-off effects of EMU entry, particularly increased financial integration and interest rate convergence.

[4]We do not include Luxembourg in our analysis due to difficulties with data availability.

[5]We do not include Greece in this latter group since it entered the euro area only in 2001, some two years later than the other countries. We have replicated the analysis including Greece in the converging group. We find that the conclusions derived in this section are not sensitive to the exclusion of Greece. Details are available on request from the authors. In addition, we do not include Finland in either group. This reflects the fact that it only joined the EU in 1994 and the evolution of its economy over part of the period under review is heavily distorted by special factors (such as the recovery from the sharp recession recorded in the early 1990s).

shock, the key macro variables in both groups would have followed identical trends. In our context, the difference-in-difference estimate of the differential effect of interest rate convergence is obtained from the following regression on annual data over the period 1995 to 2005:

$$y_{i,t} = \alpha_0 + \alpha_1 D_t + \alpha_2 G_i + \alpha_3 D_t * G_i$$

Here y_{it} denotes an observation on the value of a variable of interest in country i in period t (e.g. the current account-GDP ratio of Germany in 1997). D_t is a dummy variable taking on a value of 1 for the EMU period (t>1998) and 0 for the pre-EMU period. G_i is a dummy variable taking on a value of 1 if the country i is a member of the converging group and zero for the core group. If entry into the euro area was not associated with difference in the behaviour of the macroeconomic variables in the two country groups, then we would expect the coefficient α_3 to be not signficantly different from zero. An estimate of α_3 significantly different from zero is consistent with interest rate convergence being associated with differential behaviour across the country groups. Estimates of α_3 for a number of key macroeconomic variables and their associated t-ratios are presented in Table 1.

Clearly the sample size in this exercise is small, so the usual caveats apply both to the graphical and to the difference-in-difference estimates. Nonetheless, we believe that taking this perspective on the data does yield some useful insights into the macroeconomic effects of the interest rate convergence process.

Details on the sources of the data used in the analysis are presented in the Appendix I.

We focus on five key stylised facts which result from this analysis.[6]

As is clear from inspection of Charts 1 to 3 and also from the first three rows of Table 1 nominal and real interest rates declined in converging countries relative to core countries.

We may summarize these remarks as our first stylized fact (see Charts 1,2, and 3):

(1) There was a sizeable convergence of interest rates between the two groups of countries.

We will take interest rate convergence as the impulse triggering subsequent adjustment.

Turning to output, over the whole period, GDP growth was around 1 percentage point higher in the converging countries than in the core group. However, growth differentials tended to narrow to levels which were low by historical standards. On the surface, this

[6]The choice of these facts reflects our earlier work (Fagan and Gaspar (2007)). In that paper, we documented a number of additional facts not discussed here. In particular, we found that while there were differences in non-housing investment across the two country groups, these were not as prominent in the case of housing investment and consumption. Second, on average there was little difference in the paths of the fiscal deficits between the two country groups.

evidence suggests that the process of interest rate convergence has had little impact on output growth differentials (Chart 4). This result is confirmed by the difference-in-difference analysis: the estimated coefficient has a negative sign and it is insignificant. The result is also confirmed by a number of studies of growth differentials in the euro area (see, for example, Benalal, del Hoyo, Pierluigi, and Vidalis (2006) and ECB (2007) for a review of the evidence on output growth differentials within the euro area). In any case, the absence of statistically significant effects on overall growth differentials suggests that it is justifiable to look at intertemporal adjustment patterns in endowment economy models. It is interesting to note en passant that the differential impact of euro area participation, across the two groups of countries, is significant for housing investment, while for non-housing investment it is not (Table 1). We thus have our second stylised fact:

(2) There is no evidence of significant effects on GDP growth differentials.

There is a substantial difference in the behaviour of household expenditure across the two groups of countries. Compared to the core countries, the household savings ratio fell more rapidly in the converging countries (Chart 5), althought this effect is not significant. In addition, housing investment (as a percent of GDP) rose in the converging countries while falling in the core countries (Chart 6) , and this difference is found to be statistically significant. This differential pattern in household expenditures is clearly reflected in the different pattern in household net lending which declined sharply in the converging countries compared to the core countries (Chart 7), a phenomenon also found to be statistically significant. From Chart 8 it is also apparent the the household debt ratio of converging countries increased rapidly, approaching the levels prevailing in the core countries. Household debt also increased in the latter goup of countries, albeit at a much slower pace.

(3) Household expenditures behaved very differently in the two groups of countries. In relative terms, converging countries experienced a boom in expenditures leading to a decline in their net lending, and a more rapid buildup in debt, relative to the core countries.

Given the sharp increase in private domestic expenditure in converging countries, relative to historical trends and the muted response of output it is not surprising that a visible change occured in the external balance of these countries. As can be seen in Chart 10 net foreign assets in the core countries exhibited a mild upward trend over the period, consistent with the moderate increase in their current account surplus shown in Chart 9. In contrast the current account of converging countries moved towards a significant deficit. Such trend, in turn, led to the accumulation of a significant foreign debtor position (Chart 6). This evidence is consistent with the work of Blanchard and Giavazzi (2002) who also found that the increased financial integration in Europe, including the creation of the euro area, was associated with increasing dispersion of current account balances.The

visual impression is confirmed by the difference-in-differences estimator (Table 1) allowing us to conclude that:

(4) Current accounts balances behaved very differently across the two country groups, which core countries experiencing a broadly stable position over the period while convergence countries experience a sharp deterioration. This lead to divergent patterns in the accumulation of net foreign assets.

Chart 12 shows clear relative consumer price convergence of converging countries towards levels prevailing in core countries. Again the difference in difference estimates confirm that the real appreaciation (measured by relative consumer prices) is significant (Table 1):

(5) Converging countries recorded real appreciation vis-a-vis the core group as captured, for example, by the relative behaviour of consumer prices.

From this brief review of the stylised facts four striking features emerge regarding the different behaviour of the two groups of countries following the creation of the monetary union. First, contrary to what one might expect, there appears to have been little noticeable or statistically significant effect on output growth differentials (or non-housing investment). Second, the pattern of household spending and lending is strikingly different across the two groups of countries, with converging countries experiencing a strong boom in domestic expenditures (and household indebtness) compared to the core group. Third, reflecting the previous two facts, the current account balances and accumulation of net foreign assets behaved very differently with converging countries experiencing a marked deterioration in their current accounts balances compared to a broadly stable situation in the core group. Fourth, the behaviour of intra-euro area real exchange rates in striking, with the converging countries experiencing a notable real exchange rate appreciation vis-a-vis the core group.

Is it possible to explain these facts regarding the different behaviour between the two groups using a two-country intertemporal general equilibrium model? And what are the welfare implications for the respective countries. In the reminder of the paper paper we address both of these questions.

3 The two country general equilibrium model

We model the macroeconomic effects of financial integration using a two-country, two good endowment economy model with household sectors modelled along the lines of Blanchard (1985) and Yaari (1965). Our choice of this framework reflects our reading of the stylised facts given above.

First, up to now output growth differentials do not appear to be a major part of the story

of adjustment to the euro. Thus the use of an endowment setup, which obviously has the advantage of simplicity, can be justified. The analysis we perform for the endowment economy with non-traded goods is very much in the spirit of Obstfeld and Rogoff (2004). They also use a endowment economy framework, with traded and non-traded goods to discuss the real exchange rate implications of changing global expenditure patterns.

Second, movements in real exchange rates appear to have been an important feature of the adjustment. To tackle this issue, clearly we need a framework with (at least) two sectors.

Third, within our framework, we assume that the pre-integration difference in interest rates is due to differences in the rates of time preference across countries. This latter assumption contrasts with the main alternative approach to modelling the macroeconomic effects of financial integration, such as that employed by Gourinchas and Jeanne (2006). In that approach it is assumed that pre-integration interest rate differentials reflect differences in initial capital-output ratios and thus the process of financial integration leads to a flow of capital into the higher interest rate country which leads to an equalisation of capital-output ratios and interest rates across countries . Our approach of assuming differences in the rate of time preference across countries is motivated by three considerations. First, it is of itself interesting to examine what happens when countries characterised by a deep structural difference such as differences in the rate of time preference experience financial integration. Second, the stylised fact that the creation of the monetary union has not been associated with changes in growth differentials suggests that flows of capital and resulting changes in capital-output ratios have not been, to date, a major part of the story as far as the euro area is concerned. Third, this assumption can be seen as a simplification, which we view as proxying for a range of factors - e.g. differences in credit frictions and difference in policy credibility - which account for the initial difference in interest rates observed in the data[7].

Finally, the choice of a Blanchard-Yaari setup guarantees a well defined steady state for a financially integrated economy in the face of differences in the rate of time preference.

We consider the creation a monetary union between two (groups of) countries which, prior to the creation of the monetary union, experienced different levels of nominal and real interest rates. The first group, which we will henceforth call the home country (corresponding to the core group in the previous section), comprises the countries which, prior to the creation of the monetary union, experienced relatively low interest rates. The second group, which we will call the foreign country, consists of the former high interest rate

[7] In the context of our model, it also allows us to match two facts which are observed in the data for the euro area countries before the start of the monetary union: 1) real interest rates differed across countries and 2) net foreign assets were close to zero.

countries. We do not consider production, so we focus on an endowment economy setting. We use the H-superscript to denote home country variables and the F-superscript for foreign variables. Consumers in the home country receive a flow endowment of a tradeable good, ω^H, and a nontradeable good, η^H. Similarly, in the foreign country the consumers receive ω^F and η^F. The size of endowments are constant across time but may differ across countries, reflecting differences in country size. Both countries contain a set of Blanchard-Yaari consumers. We will assume that the death rate ρ is common across countries. We further assume that population is constant across time, and for convenience, normalised to unity in both countries, and hence the birth rate is equal to the death rate. Reflecting the data for the euro area, we allow the economies to differ in size, reflected in differences in the size of the endowments across countries. However, in the interest of simplicity, we assume that, apart from the rate of time preference, all of the remaining parameters are the same across countries.

To examine the impact of monetary union, we follow Gourinchas and Jeanne (2006). in assuming that, initially, the countries are financially autarkic, neither lending or borrowing to each other. In our setting initial steady state interest rates differ across the two countries reflecting different rates of time preference. The creation of a monetary union brings this autarkic situation to an end and the associated financial integration means that a single common interest rate prevails in the two countries from the moment of its inception[8]. This single interest rate will be an endogenous variable to be determined along with foreign assets/liabilities and consumption paths in both countries. As we will see below, the creation of a monetary union leads to a new steady state interest rate and NFA position, but also to interesting transitional dynamics.

3.1 A representative household intertemporal problem

We start by writing down the problem of a representative household of cohort t in either of the countries. Since the problem is symmetric in the home and foreign country, we do not need to employ H and F superscripts. As in all standard applications of theory, households will maximize their expected utility. In our setting, with two goods and logarithmic preferences, the household will maximize the present, expected discounted value of utility, which may be written as:

[8]In simulating the model, the creation of the monetary union is modelled as an unanticipated shock. While this is obviously a simplification, it is worth noting that the interest rate convergence which occurred in Europe happened rather quickly. Moreover, contemporary calculations based on bond yields indicate that, as late as 1996, markets assessed the probability of some converging countries entering the monetary union in 1999 as being below 50% (see, for example, Bates (1999) and Favero, Giavazzi, Iacone, and Tabellini (2000)).

$$\max_{c_t^T, c_t^N} E_t \left[\int_t^\infty \log(c_s) \exp(-\theta(s-t))\, ds \right]$$

with c_s being given by a Cobb-Douglas composite of traded and non-traded goods:

$$c_s = \kappa (c_s^T)^\alpha (c_s^N)^{1-\alpha}$$

where t and s denote time, c_t^T and c_t^N denote respectively the consumption of traded and non-traded goods, θ is the subjective discount factor while κ is a parameter used for nomalisation purposes. Since, in the model, the only uncertainty pertains to the timing of death the objective of the household may be written more simply, using our demographic assumptions to calculate expected values, as:

$$\max_{c_t^T, c_t^N} \int_t^\infty \log(c_s) \exp(-(\theta+\rho)(s-t))\, ds.$$

Each household faces a budget constraint, which we may write in the form of accumulation of assets or, if negative, debts. Specifically:

$$\frac{df_t}{dt} = (r_t + \rho) f_t + p_t^T(\omega - c_t^T) + p_t^N(\eta - c_t^N).$$

ω and η denote the flow endowments of traded and nontraded goods, respectively. The budget equation is intuitive. The individual household accumulates assets in accordance with (i) the private rate of return on past assets, given by the sum of the interest rate with the compensation for the probability of death; and (ii) the difference between the value of its endowment flows and consumption expenditure. Note that in this setting the interest rate faced by the household can vary over time.

In what follows we will let p^T be the numeraire and set it to 1. Letting z_t denote nominal consumption, i.e. $z_t = c^T + p_t^N c_t^N$, we can express the standard intratemporal first order conditions for c^T and c^N as:

$$c_t^T = \alpha z_t \tag{1}$$

and

$$p_t^N c_t^N = (1-\alpha) z_t \tag{2}$$

These expressions enable us to define a dual price index for aggregate consumption, p_t[9] so that $z_t = p_t c_t$. The budget constraint then becomes:

$$\frac{df_t}{dt} = (r_t + \rho) f_t + \omega + p_t^N \eta - z_t$$

Applying the maximum principle to solve the consumer's optimisation problem, we obtain the following Euler equation for nominal consumer expenditure:

$$\dot{z}_t = (r_t - \theta) z_t \tag{3}$$

Solving this Euler equation subject to the budget constraint gives the following "solved out" consumption function:

$$z_t = (\theta + \rho)(f_t + x_t)$$

where x_t is given by:

$$x_t = \int_t^\infty e^{-(\bar{r}(s,t)+\rho)(s-t)} \left(\omega + p_{t+s}^N \eta\right) ds.$$

Here $\bar{r}(s,t)$ denotes the average interest rate between periods t and s (see, for example, Barro and Sala-I-Martin (2003)). The consumption function tells us that current nominal consumption is a function of the discounted nominal value of lifetime resources, where the discounting takes place with respect to the sum of the nominal interest rate and the death rate. Given our demographic assumptions, the consumption function and asset accumulation equation can be aggregated across consumers. The aggregate consumption function is:

$$Z_t = (\theta + \rho)(F_t + X_t)$$

with

$$X_t = \int_t^\infty e^{-(\bar{r}(s,t)+\rho)(s-t)} \left(\omega + p_{t+s}^N \eta\right) ds.$$

The corresponding equation of motion for the aggregate foreign asset position is:

$$\frac{dF_t}{dt} = r_t F_t + \omega - \alpha Z_t$$

In deriving the latter equation we have used the first order intratemporal condition for nontraded goods together with the market clearing condition for nontraded goods: $C_t^N = \eta$.

[9] The formula for the dual price index is $p_t = \left(\frac{1}{\kappa}\right)\left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right) p_t^\alpha p_n^{(1-\alpha)}$

Note that in our setup, an endowment economy without capital and storage possibilities, economywide assets are the assets-liabilities with respect to the rest of the word. Note also, that the death rate does not appear in the aggregate asset accumulation equation. In addition, from the intratemporal first order condition for nontraded goods (2), we can show that the nontraded price is a linear function of aggregate nominal consumption:

$$p_t^N = \frac{(1-\alpha)Z_t}{\eta} \tag{4}$$

Substituting this expression into the formula for the dual price index and noting that the price of traded goods is normalised to unity, we can show that the consumer price index is a monotonically increasing function of nominal expenditure:

$$P_t = \left(\frac{1}{\kappa}\right)\left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right)\left(\frac{(1-\alpha)Z_t}{\eta}\right)^{1-\alpha} \tag{5}$$

Finally, noting that nominal consumption is given by $Z_t = P_t C_t$, we can also show that real consumption is a monotonically increasing function of total nominal expenditure.

$$C_t = \kappa\left(\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\right)\left(\frac{(1-\alpha)}{\eta}\right)^{\alpha-1} Z_t^{\alpha} \tag{6}$$

3.2 The dynamics of the aggregate economy

As noted above, the optimisation problems faced by consumers in the home and foreign countries are symmetric, so we can use the results derived in the previous section to describe the equations of motion for the two countries. Let superscript-H denote a variable for the home country and superscript-F the corresponding foreign country variable. Then using our previous results and differentiating with respect to time, we can show that the evolution of the two economies is governed by the following four equations:

$$\dot{Z}_t^H = (r_t - \theta^H)Z_t^H - \rho(\theta^H + \rho)F_t \tag{7}$$

$$\dot{Z}_t^F = (r_t - \theta^F)Z_t^F + \rho(\theta^F + \rho)F_t \tag{8}$$

$$\dot{F}_t = r_t F_t + \omega^H - \alpha Z_t^H \tag{9}$$

$$\alpha Z_t^H + \alpha Z_t^F = \omega^H + \omega^F \tag{10}$$

The first three equations are the standard in the open economy Blanchard-Yaari setup (see, for example, Blanchard and Fisher (1989)). The last equation is a condition for

equilibrium in the global market for traded goods. It makes use of the intratemporal first order condition for traded goods consumption (1) and the normalization of the price of traded goods to unity.

In what follows, we will assume, as noted above, that foreign consumers are more impatient than home consumers. This implies that their rate of time preference is higher: $\theta^F > \theta^H$. We will also use the convention that F_t measures the net foreign assets of the home consumers (a positive value means that they have positive net assets). Of course, with this convention, F_t denotes the net foreign liabilities for the foreigners.

With this apparatus in place, we are now in a position to examine the behaviour of the two economies. We start with an initial condition of financial autarky and then look at what happens following the creation of the monetary union, reflected in the common interest rate across both countries.

4 Financial autarky

Under autarky, $F_t = 0$ for all t, i.e. no international borrowing and lending takes place. We see straight away that under these conditions equations (7) and (8) imply that, in the autarkic steady state:

$$r^H = \theta^H$$

and

$$r^F = \theta^F$$

Thus the interest rates are different across countries, with $r^F > r^H$. Under autarky, the consumers in each country consume their endowments of traded and nontraded goods in each period, foreign lending and borrowing is zero and interest rates differ across countries.

5 Monetary Union

Financial integration due to monetary union implies that a common interest rate applies across the area:

$$r_t^H = r_t^F = r_t$$

where r_t is the area-wide interest rate. Note that r_t is now an endogenous variable and that - at least in the transition to the steady state - it will vary over time, hence the t-subscript.

The four equations (7) to (10) together with the initial condition, $F_0 = 0$ and the transver-

sality conditions determine the time paths of the four endogenous variables $\{Z_t^H, Z_t^F, F_t, r_t\}$. Differentiating (10) with respect to time implies:

$$\dot{Z}_t^H + \dot{Z}_t^F = 0 \tag{11}$$

Adding (7) to (8) and using (11), gives us after some rearrangement:

$$r_t = \theta^H \left(\frac{\alpha Z_t^H}{\omega^H + \omega^F} \right) + \theta^F \left(\frac{\alpha Z_t^F}{\omega^H + \omega^F} \right) + \rho(\theta^H - \theta^F) \left(\frac{\alpha F_t}{\omega^H + \omega^F} \right) \tag{12}$$

Thus, at every point in time, the (time-varying) interest rate is equal to a weighted average of the rates of time preference at home and abroad, with the weights being given by the respective shares in world consumption of traded goods, plus a term in net foreign assets. It is worthwhile to interpret the result carefully. The interest rate must be compatible with unchanging world consumption of tradable goods. It is useful to start disregarding the last term in the equation. Such a simplification would be exact if either net foreign assets would be zero, the birth (and death rate) would be nil or $\theta^F = \theta^H$. Hence, under such conditions, the interest rate would be a weighted average of the subjective rates, with weights given by the respective weights of world expenditure (as stated above). However, our simplifying assumptions don't hold true in general. Why does it make a difference? In the Blanchard-Yaari set-up the expenditure profile, over time, is the same for all households. Nevertheless, the aggregate time profile is affected by the arrival of newborns. They affect average expenditure because, by assumption, they are born without assets (or debts). Therefore, they have a lower (higher) expenditure level than the average in the economy. The difference, relative to the respective average, is $-(\theta^H + \rho)F_t$ for the home country newborns and $(\theta^F + \rho)F_t$ for the foreign country newborns, since $(\theta^i + \rho), i = H, F$ is the marginal propensity to consume out of wealth. In both cases their weight in the overall population is given by the birth rate ρ. The impact on expenditure is, thus, $-\rho(\theta^H - \theta^F)\alpha F_t$. Since $\theta^F > \theta^H$ the term within brackets is negative. As F_t increases over time, the equilibrium interest rate becomes lower than the consumption-weighted average of the rates of time preference.

From (12)we see straight away that the initial interest rate, r_0, following the creation of the monetary union, must lie between the interest rates which prevailed in the two countries. We see this by noting that at this point, let us say $t = 0$, we have by assumption $F_0 = 0$. The equation then implies that $\theta^H < r_0 < \theta^F$.

Using (10) to eliminate Z_t^F from (12) yields:

$$Z_t^H = \left(\frac{1}{\alpha}\right)\left(\frac{r_t - \theta^F}{\theta^H - \theta^F}\right)(\omega^H + \omega^F) - \rho F_t \tag{13}$$

The partial derivative of this expression with respect to r is:

$$\frac{dZ^H}{dr_t} = \frac{1}{\alpha(\theta^H - \theta^F)} < 0 \tag{14}$$

and, by a similar sequence of steps,

$$Z_t^F = \left(\frac{1}{\alpha}\right)\left(\frac{\theta^H - r_t}{\theta^H - \theta^F}\right)(\omega^H + \omega^F) + \rho F_t \tag{15}$$

These equations show us that once we know the paths F and r, we can determine the paths of nominal consumption in both countries. Once we know the paths of $Z^H$and Z^Fwe can also derive for each country paths for nontraded goods prices, the consumer price indices and real consumption using the expressions (4),(5) and (6) given earlier.

By a process of repeated substitution in (7) to (10), we can reduce the model to a pair of (nonlinear) differential equations in F and r.

$$\dot{F}_t = (r_t + \alpha\rho)F_t + \omega^H - \left(\frac{r_t - \theta^F}{\theta^H - \theta^F}\right)(\omega^H + \omega^F) \tag{16}$$

$$\dot{r}_t = \frac{\alpha\rho\omega^H(\theta^H - \theta^F)}{(\omega^H + \omega^F)} + (r_t - \theta^H - \alpha\rho)(r_t - \theta^F) - \frac{\alpha(\theta^H - \theta^F)\rho^2(1-\alpha)}{(\omega^H + \omega^F)}F_t \tag{17}$$

F is a predetermined variable while r is a jump variable. The pair of equations can be solved numerically by using standard two-point boundary techniques.

5.1 The steady state of the monetary union

Although our primary interest is in the shorter term dynamic adjustment to the creation of the monetary union, we present in this section the key features of the (post Monetary Union) steady state. This is given by the fixed point of equations (16) and (17). Define the weighted average of the rates of time preference (r^{av}) with the weights being given by the relative sizes of the traded endowment bundles as :

$$r^{av} = \frac{\omega^H\theta^H + \omega^F\theta^F}{\omega^H + \omega^F}$$

Now, letting a bar over a variable denote the steady state, we show in Appendix II that:

$$\theta^H < \bar{r} < r^{av} < \theta^F$$

and

$$\bar{F} > 0$$

In words, the steady state interest rate lies between the interest rates that prevailed in the home and foreign countries prior the the monetary union. In fact the interval is narrower: the steady state interest rate is lower than an endowment weighted average of the initial interest rates. In addition, the net foreign asset position of the home country is positive, compared with the initial value of zero, implying that it becomes a net creditor. In the appendix, we also establish the following facts about the post monetary union steady state relative to the initial autarkic steady state:

1. Both nominal and real consumption in the home country are higher;
2. Both nominal and real consumption in the foreign country are lower;
3. The price of nontraded goods in the home country is higher and (since the price of traded goods is the numeraire) the overall consumer price index is higher.
4. The price of nontraded goods in the foreign country is lower as is the consumer price index.

5.2 Transition paths

To explore the dynamics, we linearise (16) and (17) around the new steady state to obtain:

$$\dot{\tilde{F}}_t = (\bar{r} + \alpha\rho)\tilde{F}_t + \left(\bar{F} - \frac{\omega^H + \omega^F}{\theta^H - \theta^F}\right)\tilde{r}_t \tag{18}$$

$$\dot{\tilde{r}}_t = -\left(\frac{\alpha(\theta^H - \theta^F)\rho^2(1-\alpha)}{\omega^H + \omega^F}\right)\tilde{F}_t + (2\bar{r} - \rho - \theta^H - \theta^F)\tilde{r}_t \tag{19}$$

where a bar over a variable denotes the steady state value while a tilde denotes a deviation from steady state. Saddle path stability requires that this system has one root with positive real parts and one root with negative real parts. This requires that the determinant of the associated matrix is negative. A sufficient condition for this to be the case is that $\bar{r} < \frac{\theta^H + \theta^F + \rho}{2}$, which is likely to be the case in practice. We will assume that this condition is always satisfied for the relevant range of parameters we are interested in (and of course

we verify this in numerical implementations). The phase diagram of the system is presented in Figure 1.

We note that the locus $\dot{F} = 0$ is downward sloping in F-r space and it can easily be shown that the saddle path is less steeply sloped[10]. The $\dot{r} = 0$ locus is upward sloping and, for reasonable parameter values, almost a vertical line.

Following the creation of the monetary union, the interest rate jumps to a new level lying between the interest rates that prevailed in the two countries prior to the union. Since the level of foreign assets is predetermined, the economy jumps to the point (A) on the saddle path associated with the initial interest rate r_0.

We can now establish that this initial interest rate is higher then the weighted average rate r^{av}. To see this, note from (16), that if r_0 were equal to r^{av}, given the initial value of $F_0 = 0$, then $\dot{F}$ would be zero. This corresponds to point (C) on the phase diagram. At this point, countries would be neither lending to (or borrowing from) each other. Consumption in both countries would be equal to pre-EMU levels, with consumers in each country consuming their endowments. This point cannot be an equilibrium since it does not lie on the saddle path. The equilibrium point on the saddle path (point (A)) lies to the right of it. This gives us one key result for the dynamics. Initially, following the creation of the EMU, the common interest rate lies above the weighted average of the rates of time preference.

Thereafter, foreign assets/debt rises gradually while the interest rate falls as the economy converges to its new steady state (B), characterised by a positive level of foreign assets for the home country and $r < r^{av}$as described in Section 5.1.

The time paths of the remaining variables can be derived in a straightforward manner. It is easy to establish that nominal consumption in the home country jumps down on impact. To see this, note that (1) at $r_0 = r^{av}$, nominal consumption would equal the pre-EMU level; (2) in fact, as shown above, $r_0 > r^{av}$; and (3) the partial derivative of Z^H with respect to r_t is negative, as shown in (14). These three facts imply that the initial level of home consumption following the monetary union is lower than the pre-EMU level. The same applies to real consumption, the nontraded price and the consumer price index since all three of these variables are increasing in Z^H, see (4) to (6). In particular, there is a decline in the real exchange rate, on impact, in line with the stylized facts reported in Section 2. All of these variables then monotonically increase to new steady state levels, higher than the initial (pre-EMU) steady state.

The corresponding variables for the foreign country mirror those of the home country but with the opposite sign. Nominal and real consumption in the foreign country jump up

[10]Details available from the authors on request.

on impact and then gradually decline to new, lower, steady state levels. Prices also move accordingly. In particular, there is an appreciation of the real exchange rate on impact.

With total resources fixed by our endowment assumption, the divergent shifts in domestic demand give rise to movements in the current account balance and set in train an accumulation of foreign assets/liabilities. The home country moves into surplus (and the foreign country into deficit). These current account imbalances are gradually eliminated as the economies converge to the new steady state foreign asset/debt ratios. At the same time, the divergent movements in prices implies that initially the foreign country initially experiences a real appreciation before gradually converging to a steady state characterised by a lower real exchange rate.

To summarize, our model predicts the following features following the creation of the monetary union. First, there are divergent movements in nominal and real consumption in the two areas, with the foreign country experiencing an initial boom compared to the home country. Second, this movement in demand leads to the emergence of current account imbalances, with the foreign country experiencing a deficit vis-a-vis the home country. Finally, the foreign country experiences an initial real appreciation vis-a-vis the home country. In qualitative terms, this is fully in line with the stylised facts for the euro area countries outlined in Section 2.

6 A numerical example

In order to further illustrate the workings of the model, we now present the results of a numerical simulation of the model.[11] We consider the case of two (groups of) countries: the core (home) and converging (foreign) countries. We calibrate the model as follows. $\omega^H$is set to 0.65, the share of the core countries in euro area GDP while $\omega^F$is set to 0.35. We assume that the share of traded consumption in total consumption (α) is 0.33 in both countries, consistent with available input-output evidence for the euro area. The traded goods price is the numeraire in both counties and is set to unity. We also assume for convenience that the initial nontraded price in both countries is unity. This then gives us implied values for the nontraded goods endowments η^H and η^F. The mortality rate, ρ, is set to 0.02 per annum, corresponding to a life expectancy of 50 years, a standard value in these models. Given our assumption of constant population, the mortality rate is equal to the birth rate. $\theta^H$is set to 0.04 while $\theta^F$is set at 0.05. Initially, under autarky, these parameters imply that the interest rates in home and foreign are 4% and 5% respectively. In the initial steady state, consumption in both countries is thus equal to the country-specific

[11] To solve the nonlinear differential equations we use the standard Matlab function (BVP4C)

endowments.

The response of the variables to the creation of a monetary union is shown in Figures 2 which (except for interest rates, current account balances and net foreign assets) shows the percentage difference from the pre-EMU steady state. On impact, the interest rate jumps to a common level of 4.4%. Real consumption in the core countries jumps down by 2% on impact while consumption in the converging countries jumps up by 4%. These shifts in relative domestic demand are associated with the emergence of current account imbalances with the current account balance of the core countries moving to an initial surplus of 2 percent of GDP (and an associated deficit in the converging countries of 4 percent of GDP - the differences in the numbers reflecting differences in the size of the economies). Thereafter, as the converging countries accumulate foreign debt, their consumption declines gradually to a new level some 15% below the autarkic steady state value. Correspondingly consumption in the core countries rises to a new steady-state level, higher than the autarkic value. The patterns of relative demand are associated with notable movements in cross-country relative prices. Nontraded prices in the converging countries jump up by 12% while in the home countries, nontraded prices fall by 7% on impact. The real exchange rate of the converging countries - measured by relative consumer prices - experiences a sharp initial appreciation of the order of 20%. Thereafter, there is a slow convergence of prices to the new steady state, characterised by higher nontraded prices in the home country and lower nontraded prices in the converging countries. The common interest rate gradually rises to a new steady state value, lying, as demonstrated in the previous section, below the weighted average of the two autarkic rates. Overall, since the graphs are in annual terms, it is clear that the convergence to the steady state is very slow.

7 Some Welfare Implications

In the overlapping generations setup which we have employed here, the computation of aggregate welfare and changes therein is more complicated than in the usual case of infinitely-lived agents. This stems from the fact that, in order to derive a social welfare function, the welfare of the different past and future generations need to be aggregated using some weighting scheme. One prominent example of such a scheme is the time-consistent approach of Calvo and Obstfeld (1988). As it happens, in the context of the example considered in this paper, we can derive strong welfare conclusions without recourse to the device of constructing such a social welfare function.

In fact, it can be easily established that the welfare of all currently alive and future generations in both countries rises (or more specifically does not decline) as a result of the

creation of the monetary union. This can be seen by means of the following 'revealed-preference' argument. Given our assumptions, set out earlier, in the pre-EMU steady state each agent in the economy consumes his own endowment bundle in each period and country and agents do not engage in borrowing or lending. Now, following the creation of the monetary union, this consumption path - consuming the endowment each period - remains feasible. However, agents in fact choose different paths for consumption. Since the agents are choosing consumption paths to maximise their welfare and they have perfect foresight, we can conclude that the introduction of the monetary union does not reduce the welfare of any currently alive or future generation compared with the pre-EMU situation. In short, the creation of the monetary union and the associated financial integration expands the opportunity set of all agents in the economy. This enables agents to derive gains from the possibility to engage intertemporal trade (which previously was not available) and thus leads to an improvment in (or at least no loss of) welfare. This is a standard result, which is, for example, discussed in detail in Chapter 1 of Obstfeld and Rogoff (1996). As we shall see below, while all generations gain from monetary union, the extent of the gain differs across generations and across countries.

In order to illustrate this point numerically, we compute for different representative generations in each country, the welfare that obtains under EMU compared to the welfare for the same agent that would prevail under financial autarky. For this purpose, we use the same parameters which were employed in the previous section. Specifically, for an agent born in period v, we solve solve for the consumption path and calculate their welfare at the time of their birth, given by:

$$W_v = \int_v^{\infty} log(c_{v,s}) e^{-(\theta+\rho)(s-v)} ds \tag{20}$$

where $c_{v,s}$ denotes the consumption of an agent born in period v in period t.

In the autarkic steady state, consumption is equal to the (constant) endowment of traded and non-traded goods each period, the welfare of the home and foreign consumer is given by calculating the integral in expression (20). This gives, for the home and foreign consumer respectively:

$$W_V^{H,A} = \frac{log(\omega^H + \eta^H)}{\theta^H + \rho}$$

and

$$W_V^{F,A} = \frac{log(\omega^F + \eta^F)}{\theta^F + \rho}$$

Note that under our assumptions, within each country welfare is the same across generations under autarky. This is our baseline case.

Now we consider what happens when, from period $t = 0$ onwards, a monetary union exists. For any generation, alive at time 0 or born subsequently, in either country we can calculate welfare be deriving the time path for the generation's consumption and calculating the value of the integral in (20)[12]. We repeat this process for different generations in the two countries.

Before presenting the results, it is important to note that given our assumptions about demographics and consumption in the pre-EMU state, the optimisation problem facing agents of generation 0 will be identical to that of all previous generations currently alive in the economy in period 0. This reflects the fact that they all have the same life expectancy, the same endowment bundle and the same initial stock of financial assets (namely zero). Thus the results for this generation will also show us the welfare gains for all individuals alive at the time when the monetary union was created.

In order to present the results in an easily interpretable form, we express the gains in welfare in terms of the equivalent percentage difference in annual consumption necessary to yield the same level of welfare in the pre and post-EMU situations. This is given by:

$$(\theta + \rho)\left(W_{EMU} - W_A\right)$$

The results of this exercise are presented in Table 2, which shows results for the generations alive on the date of the creation of the monetary union (denoted $-\infty$ to 0 in the Table) and 5, 10, 20, 50 and 100 years afterwards. The row labelled ∞ refers to the welfare gains of an individual born in the steady state of the monetary union. The table shows us the percentage increase in annual consumption which would be needed without EMU in order to yield the same level of lifetime utility to the agent as under EMU. For example, for the generation of agents born in date 0 in the foreign country, the lifetime gain in welfare is equivalent to a permanent increase of 0.19% in consumption.

[12]The time path of the individual consumption depends on aggregate variables (the interest rate and the price of non-traded goods). To derive a functional expression for consumption, we substitute the aggregate variables in the individual consumer's problem by using the linear approximations given by the solution of the linearised model. Solving the consumer's problem with this this substitution then gives us an expression for individual's consumption as function of time. Standard numerical integration algorithms can then be employed to calculate welfare. In fact, we calculate the integral using Gaussian quadrature with the standard Matlab function quadl.

We oberve the following features. First, welfare of all generations in both countries increases as a result of the creation of the monetary union, in line with the revealed preference argument given above. Second, welfare gains tend to be larger in the foreign than in the home country. Third, the gains in welfare are larger for later generations and become very sizeable for the later generations in the foreign country. The latter two features deserve some comment. Intuitively, the size of the welfare gains for any generation depends on how large the difference in the 'average' real interest rate facing a typical member of that generation over its expected lifetime is from what would have prevailed without the EMU. By way of illustration, the last two columns in Table 2 show the average real interest rate expected to prevail over the lifetime of different generations[13]. From the results in the previous sections, we know that the gap between the actual real interest rate and the pre-EMU interest rates is larger for the foreign than the home country. (This of course partly reflects our assumptions on the relative size of the two countries.) Therefore, for any given generation, the welfare gains will be larger for foreign rather than the home country.

Let us now consider explicitly the influence of consumer inflation on the real interest rate in each country. In the foreign country the real interest rate will be declining over time. The decline results from both a fall in the nominal interest rate and an increase in inflation (or, more precisely, a gradual fall to zero in the rate of deflation). Hence the gap with the pre-EMU rate will increase over time. This implies that generations in the foreign country which are born later will experience larger welfare gains than generations born earlier. For the home country, the real interest rate is rising slightly over time (reflecting the offsetting effect of a falling nominal interest rate and declining inflation, with the latter dominating). Hence the gap with the pre-EMU real rate will also increase for the home country. Therefore, also here, generations born later will experience higher welfare gains than earlier generations[14].

8 Conclusion

The move to monetary union led to interest rate convergence among participating countries. Interestingly, this convergence was associated with sharp differences in the behaviour of macroeconomic variables between two groups of countries: core countries (Germany and

[13]The expected real interest rate over the life of a generation born in period t is calculated as $\rho \int_t^{\infty} e^{-\rho(s-t)}(r(s)-\pi(s))ds$ where r(s) and $\pi(s)$ denote respectively the nominal interest rate and consumer price inflation rate in period s and ρ is the mortality rate.

[14]In our setting, as discussed in Section 5.2, there is a discontinuous decrease (increase) in consumer prices in the home (foreign) country. If we would have considered habit formation (as in Fagan and Gaspar (2007)), the dynamics of the real exchange rate would include an initial gradual adjustment and a subsequent inversion.

countries closely associated with Germany, such as Belgim, the Netherlands, France, Luxembourg and Austria) and converging countries (Spain, Ireland, Italy and Portugal). The difference is that the former group enjoyed relatively low interest rates prior to monetary unification.

We documented a number of key stylized facts regarding the macroeconomic effects of this interest rate convergence. We then proceeded to develop a a simple two-country, two-good (traded and non-traded) dynamic general equilibrium model to explain these facts. We used this model to derive a number of normative implications regarding the welfare effects of the monetary union.

We showed that there were notable cross-country differences in the behaviour of key macroeconomic aggregates: domestic demand, current account balances and intra-euro area real exchange rates. Compared to the core countries, the converging countries experienced a boom in domestic demand (principally reflecting higher household expenditures), a deterioration of the current account and an appreciation of the real exchange rate. Interestingly, the convergence of interest rates does not seem to have been associated with significant changes in output growth differentials.

The simple model that we develop is capable of capturing most salient, qualitative features of adjustment. Specifically, in the two country, two goods endowment economy, the convergence in interest rates resulting from the monetary union leads to notable differences in expenditures in the two country groups, with former high interest rate countries experiencing a sharp increase in household expenditure, accompanied by a current account deficit and the accumulation of a debtor position vis-a-vis the remaining countries. This is followed by a slow process of adjustment to a new steady state in which consumption is lower (higher) in the high interest rate countries (low interest rate countries) reflecting the accumulation of foreign assets/liabilities. In our model, the real exchange rate (defined as the relative price of non-tradeable goods) follows exactly the path of aggregate expenditure in both countries. In other words, high interest rate countries initially experience an upward jump in the real exchange rate followed by a gradual decline to a level lower than in the initial steady state.

As regards welfare, we presented a very simple an intuitive revealed preference argument to show that, in our setup, monetary unification is welfare improving for all generations in both countries. The argument shows that monetary unification leads to an expansion of the opportunity set of all generations. It leads to an allocation of resources which is Pareto superior. While all generations experience a gain in welfare, the extent of the gains differs across countries and across generations. In our numerical example, the gains in welfare are larger in the former high interest rate countries and, within the same country,

later generations tend to benefit more than generations born earlier. In line with standard arguments regarding the gains from intemporal trade, this pattern can be explained by the fact that, compared to a non-EMU situation, the difference in the real interest rate facing countries and generations is larger for the foreign country and, within a given country, in higher for later generations.

Appendix I: Data sources

The main data source for the results presented in Section 2 is the EU Commission's AMECO database. The codes for the series employed are as follows: nominal short-term interest rates (ISN), hnominal long-term interest rates (ILN), household savings rate (ASGH), nominal housing investment (UIGDW), current account balance (UBCA), HICP (ZCPIH), whole economy net lending (UBLA), general government net lending (UBLG), household net lending (UBLH), nominal GDP (UVGD) andreal GDP (OVGD). Non housing investment is calculated as the difference between total investment (UIGT) and housing investment.

The exceptions to the use of AMECO are as follows. Credit to households data came from the OECD Financial Balance Sheet databank (except in the case of Ireland where the the source was various issues of the Central Bank of Ireland Quarterly Bulletin). Data on net foreign assets are taken from Lane and Milesi-Ferreti (2006). Data on the Harmonised Index of Consumer Prices (HICP) were taken from Eurostat while the data on relative consumer prices come from Eurostat's Comparative Consumer Price Level dataset and relate to price levels for final consumption by private households. For Portugal and Ireland, the data on net lending shown in Chart 13 are from the Banco de Portugal and the authors own calculations respectively.

Ratios to GDP were calculated by dividing the respective series (as described above) by nominal GDP. The variables for core countries were calculated by taking cross-country arithmetic means of the respective variables for Belgium, Germany, France, Austria and the Netherlands. Converging country variables were similarly calculated using data for Ireland, Italy, Portugal and Spain.

Appendix II: Steady state

To determine the post monetary union steady state of we proceed by first deriving the steady state foreign assets as a function of r and then, using these results, determining the

steady state value of r. In what follows, let $r^{av} = \frac{\omega^H \theta^H + \omega^F \theta^F}{\omega^H + \omega^F}$, i.e. the weighted average of the rates of time preference in the two countries. As regards the first step, consider the fixed point of (9) as a function of the interest rate, i.e.

$$F(r) = \left(-\omega^H + \left(\frac{r - \theta^F}{\theta^H - \theta^F} \right) (\omega^H + \omega^F) \right) / (r + \alpha\rho) \tag{21}$$

This is clearly a continuous function for $r \neq -\alpha\rho$. Moreover, for $r < \theta^F$ this is a decreasing function of r. Now,

$$F(\theta^H) = \frac{\omega^F}{r + \alpha\rho} > 0$$

and

$$F(r^{av}) = 0$$

We next examine the steady state interest rate. From the differential equation for the interest rate (17), the steady state of r is given by the solution of:

$$g(r) = \frac{\alpha\rho\omega^H(\theta^H - \theta^F)}{(\omega^H + \omega^F)} + (r - \theta^H - \alpha\rho)(r - \theta^F) - \frac{\alpha(\theta^H - \theta^F)\rho^2(1-\alpha)}{(\omega^H + \omega^F)} F(r) = 0 \tag{22}$$

Clearly, g(r) is a continuous function. Now, substituting for F(r) from (21), we have, for $r = \theta^H$:

$$g(\theta^H) = \frac{-\alpha\rho\omega^F(\theta^H - \theta^F)}{(\omega^H + \omega^F)} - \frac{\alpha(\theta^H - \theta^F)\rho^2(1-\alpha)}{(\omega^H + \omega^F)} \frac{\omega^F}{r + \alpha\rho} > 0 \tag{23}$$

Also:

$$g(r^{av}) = \left(r^{av} - \theta^H\right)\left(r^{av} - \theta^F\right) < 0$$

Thus, since g(r) is continuous on R, by the Intermediate Value Theorem, there exists an $\bar{r}$, with $\theta^H < \bar{r} < r^{av} < \theta^F$, such that $g(\bar{r}) = 0$. This is the steady state value of r that we are interested in.

Combining this result with the results we had for foreign assets, we obtain for this steady state, since F is a decreasing function of r and $\bar{r} < r^{av}$:

$$\bar{F} > 0$$

In order to determine the effect of interest rate convergence on steady state nominal consumption and prices, we proceed as follows. First, rearranging (7) we have, in steady state:

$$F = \left(\frac{r - \theta^H}{\rho(\theta^H + \rho)} \right) Z^H$$

Substituting into the steady-state version of (13) we get

$$Z^H = \left(\frac{r - \theta^F}{r + \rho} \right) \left(\frac{\omega^H + \omega^F}{\alpha} \right) \left(\frac{\theta^H + \rho}{\theta^H - \theta^F} \right)$$

Differentiating with respect to r we have:

$$\frac{dZ^H}{dr} = \frac{(\omega^H + \omega^F)(\theta^F + \rho)(\theta^H + \rho)}{(r + \rho)^2(\theta^H - \theta^F)\alpha} < 0$$

Recalling that $r^{av} = \frac{\omega^H \theta^H + \omega^F \theta^F}{\omega^H + \omega^F}$. We have shown above that when $r = r^{av}$, $F = 0$. At this interest rate, $\alpha Z^H = \omega^H$. But this is the level of (nominal) consumption which prevailed in the pre-EMU steady state. In the post EMU steady state, we know that:

$$\bar{r} < r^{av}$$

Given that the derivative of Z^H wrt r is negative, this implies that in the EMU steady state:

$$\bar{Z}^H > Z_0^H$$

Using the expressions given in Section 3.1, this also implies that in the EMU steady state, real consumption, the price of non-traded goods and the consumer price index are all higher than in the autarky steady state. Noting the global equilibrium condition for traded goods (10), we clearly have:

$$\bar{Z}^F < Z_0^F$$

with corresponding implications for real consumption, nontraded prices and the consumer price price index in the foreign country.

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Table 1. Difference-in-difference estimates

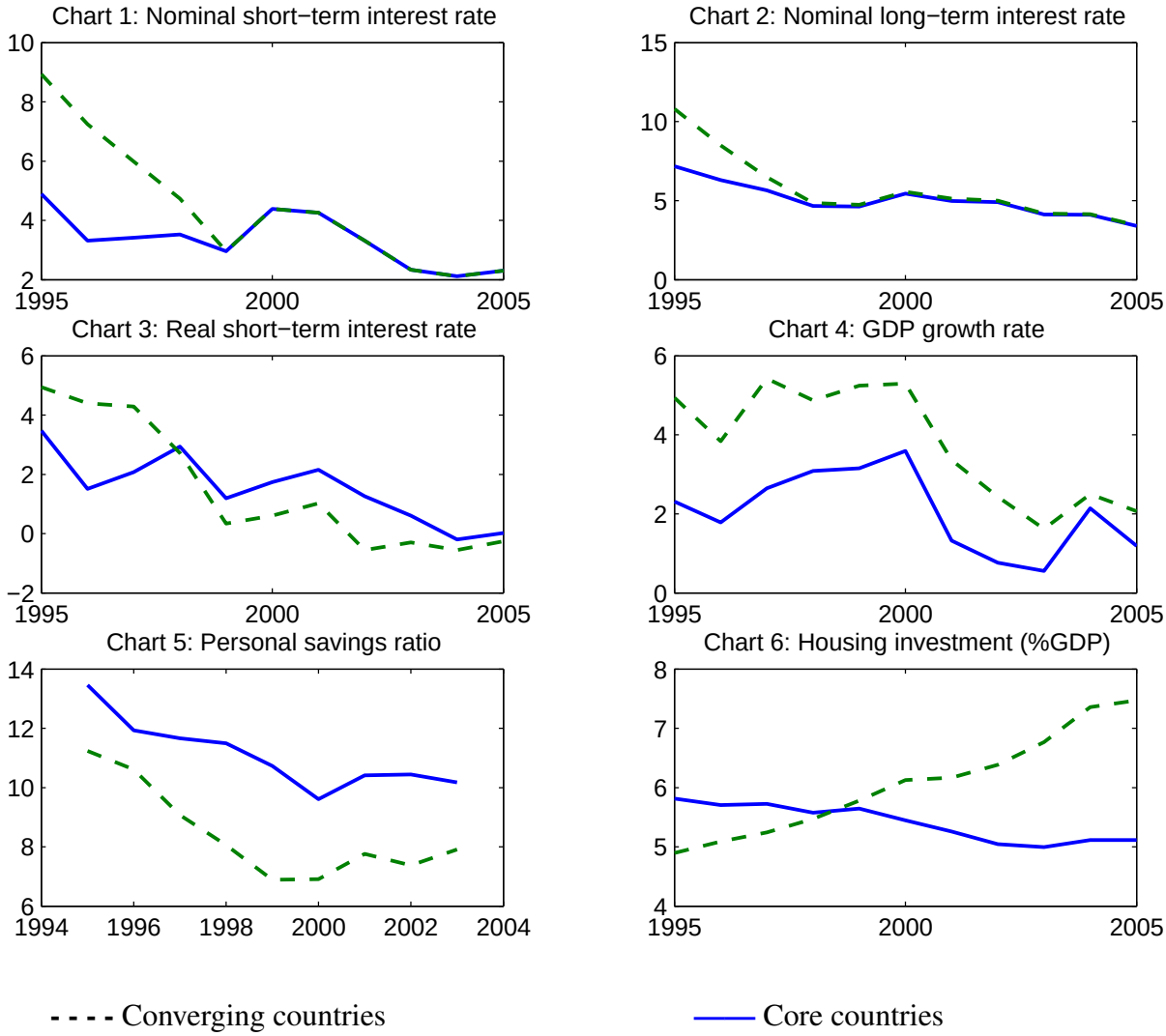
Variable	Coefficient (α_3)	t-ratio
Nominal short interest rate	-2.93	-6.3
Nominal long interest rate	-1.62	-3.2
Real short interest rate	-2.63	-6.2
Households savings ratio	-0.51	-0.4
Housing investment/GDP	1.87	2.9
Non-housing investment/GDP	0.98	1.1
GDP growth rate	-0.91	-1.0
Current account balance/GDP	-3.81	-2.8
Net foreign assets/GDP	-19.12	-2.0
HICP inflation	-0.29	-0.8
Relative consumer prices	7.92	2.2
Household net lending/GDP	-3.05	-2.69
Companies net lending/GDP	-1.91	-1.45
General Government net lending/GDP	0.63	0.71

(based on annual data for 1995 to 2005)

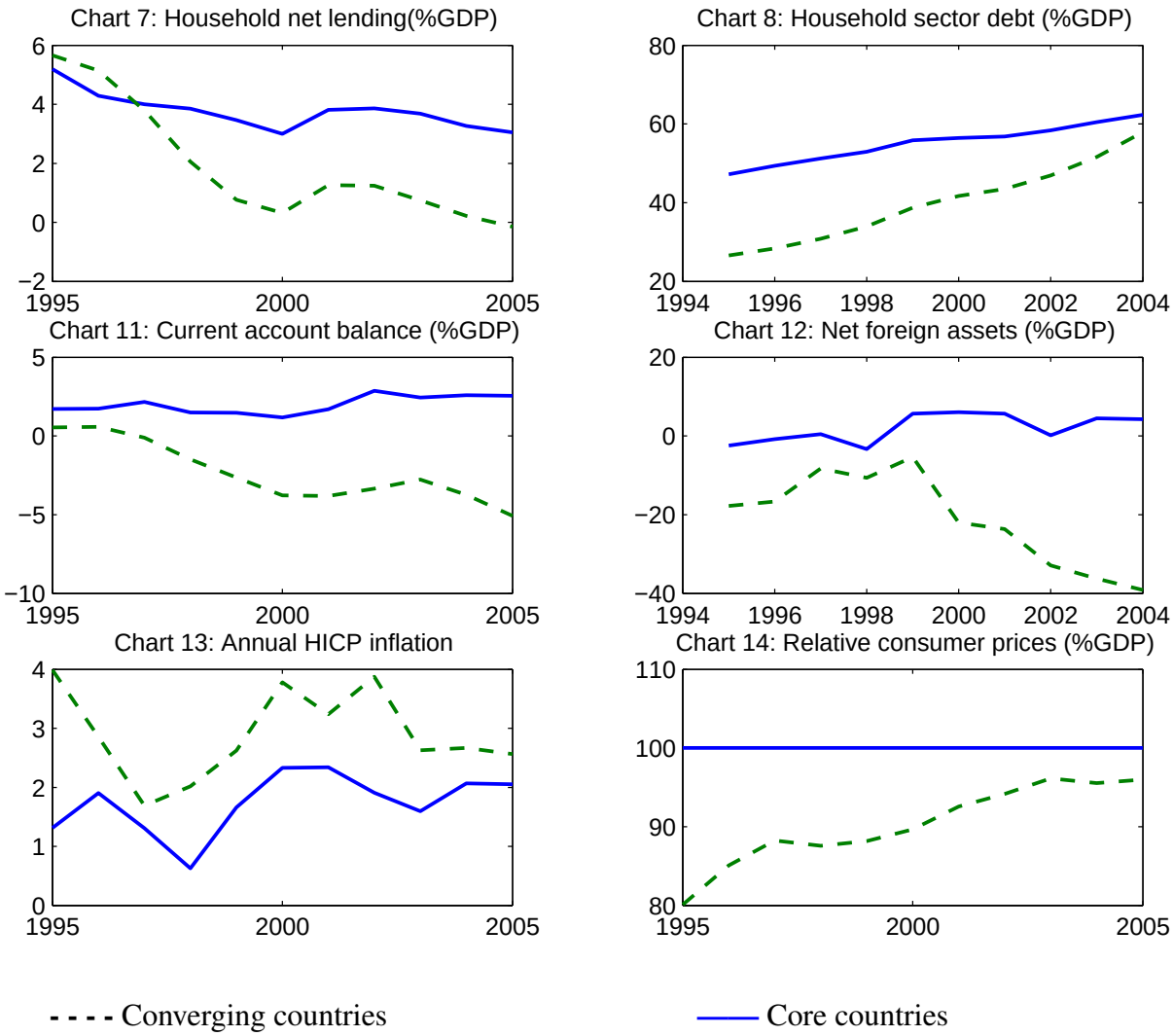
Table 2. Welfare Gains from EMU* and expected average real interest rate

	Welfare Gains		Expected Real Rate	
Generation	Home	Foreign	Home	Foreign
$-\infty$ to 0	0.05	0.19	4.14	4.66
5	0.05	0.19	4.15	4.63
10	0.05	0.22	4.15	4.61
20	0.07	0.33	4.16	4.56
50	0.17	0.95	4.17	4.45
100	0.32	2.22	4.17	4.32
∞	0.50	4.35	4.18	4.18

(* as a percent of annual consumption)



The charts display averages for each variable and country group. "Core countries" includes Austria, Belgium, France, Germany and the Netherlands. "Converging countries" includes Ireland, Italy, Portugal and Spain.



The charts display averages for each variable and country group. "Core countries" includes Austria, Belgium, France, Germany and the Netherlands. "Converging countries" includes Ireland, Italy, Portugal and Spain.

Figure 1: Adjustment dynamics for interest rates and foreign assets

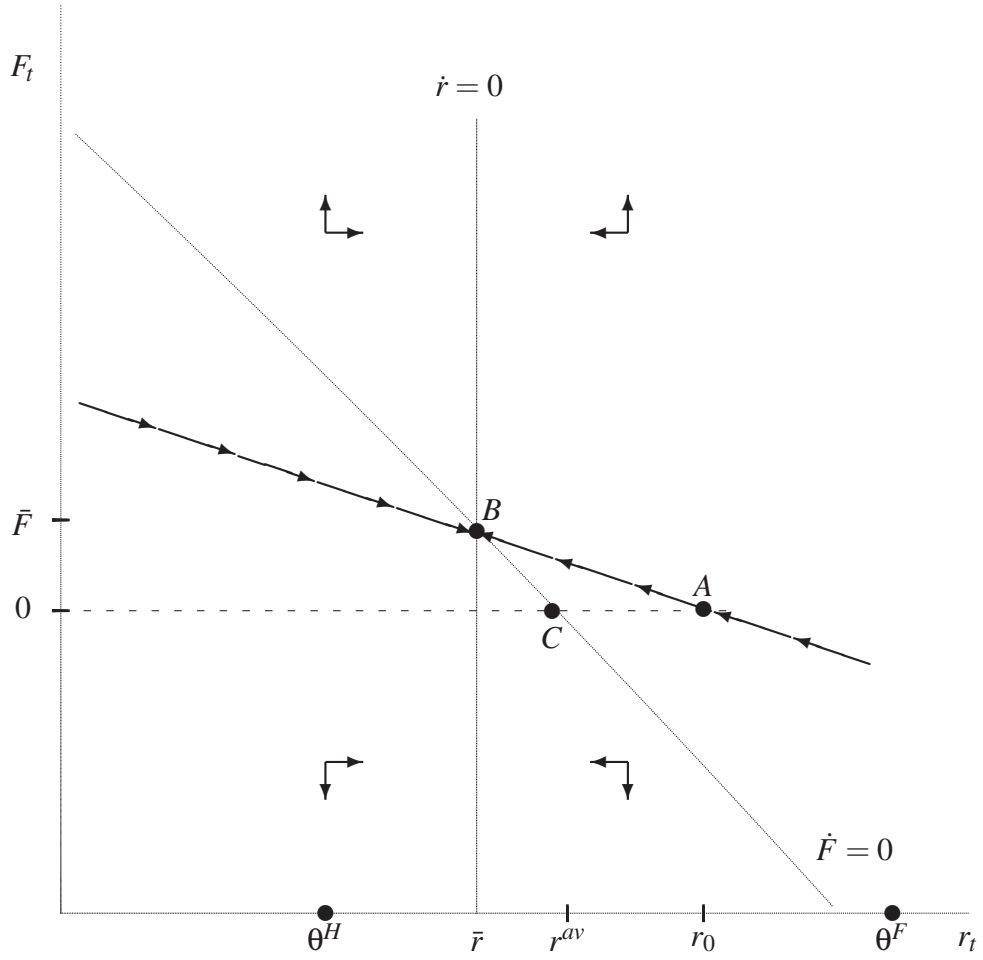


Figure 2: Simulated response to monetary union